NAG Toolbox for MATLAB

e01sg

1 Purpose

e01sg generates a two-dimensional interpolant to a set of scattered data points, using a modified Shepard method.

2 Syntax

```
[iq, rq, ifail] = e01sg(x, y, f, nw, nq, 'm', m)
```

3 Description

e01sg constructs a smooth function $Q(x,y)$ which interpolates a set of m scattered data points (x_r, y_r, f_r), for $r = 1, 2, \ldots, m$, using a modification of Shepard's method. The surface is continuous and has continuous first partial derivatives.

The basic Shepard 1968 method interpolates the input data with the weighted mean

$$Q(x,y) = \frac{\sum_{r=1}^{m} w_r(x,y) q_r}{\sum_{r=1}^{m} w_r(x,y)},$$

where $q_r = f_r$, $w_r(x,y) = \frac{1}{d_r^2}$ and $d_r^2 = (x - x_r)^2 + (y - y_r)^2$.

The basic method is global in that the interpolated value at any point depends on all the data, but this function uses a modification (see Franke and Nielson 1980 and Renka 1988c), whereby the method becomes local by adjusting each $w_r(x,y)$ to be zero outside a circle with centre (x_r, y_r) and some radius R_w. Also, to improve the performance of the basic method, each q_r above is replaced by a function $q_r(x,y)$, which is a quadratic fitted by weighted least-squares to data local to (x_r, y_r) and forced to interpolate (x_r, y_r, f_r). In this context, a point (x,y) is defined to be local to another point if it lies within some distance R_q of it. Computation of these quadratics constitutes the main work done by this function.

The efficiency of the function is further enhanced by using a cell method for nearest neighbour searching due to Bentley and Friedman 1979.

The radii R_w and R_q are chosen to be just large enough to include N_w and N_q data points, respectively, for user-supplied constants N_w and N_q. Default values of these parameters are provided by the function, and advice on alternatives is given in Section 8.2.

This function is derived from the function QSHEP2 described by Renka 1988a.

Values of the interpolant $Q(x,y)$ generated by this function, and its first partial derivatives, can subsequently be evaluated for points in the domain of the data by a call to e01sh.

4 References

Bentley J L and Friedman J H 1979 Data structures for range searching *ACM Comput. Surv.* **11** 397–409

Franke R and Nielson G 1980 Smooth interpolation of large sets of scattered data *Internat. J. Num. Methods Engrg.* **15** 1691–1704

Renka R J 1988a Algorithm 660: QSHEP2D: Quadratic Shepard method for bivariate interpolation of scattered data *ACM Trans. Math. Software* **14** 149–150

Renka R J 1988c Multivariate interpolation of large sets of scattered data *ACM Trans. Math. Software* **14** 139–148

Shepard D 1968 A two-dimensional interpolation function for irregularly spaced data *Proc. 23rd Nat. Conf. ACM* 517–523 Brandon/Systems Press Inc., Princeton

5 Parameters

5.1 Compulsory Input Parameters

1: **x(m) – double array**

2: **y(m) – double array**

The Cartesian co-ordinates of the data points (x_r, y_r), for $r = 1, 2, \ldots, m$.

Constraint: these co-ordinates must be distinct, and must not all be collinear.

3: **f(m) – double array**

$\mathbf{f}(r)$ must be set to the data value f_r, for $r = 1, 2, \ldots, m$.

4: **nw – int32 scalar**

The number N_w of data points that determines each radius of influence R_w, appearing in the definition of each of the weights w_r, for $r = 1, 2, \ldots, m$ (see Section 3). Note that R_w is different for each weight. If $\mathbf{nw} \leq 0$ the default value $\mathbf{nw} = \min(19, \mathbf{m} - 1)$ is used instead.

Constraint: $\mathbf{nw} \leq \min(40, \mathbf{m} - 1)$.

5: **nq – int32 scalar**

The number N_q of data points to be used in the least-squares fit for coefficients defining the nodal functions $q_r(x, y)$ (see Section 3). If $\mathbf{nq} \leq 0$ the default value $\mathbf{nq} = \min(13, \mathbf{m} - 1)$ is used instead.

Constraint: $\mathbf{nq} \leq 0$ or $5 \leq \mathbf{nq} \leq \min(40, \mathbf{m} - 1)$.

5.2 Optional Input Parameters

1: **m – int32 scalar**

Default: The dimension of the arrays **x**, **y**, **f**. (An error is raised if these dimensions are not equal.)

m, the number of data points.

Constraint: $\mathbf{m} \geq 6$.

5.3 Input Parameters Omitted from the MATLAB Interface

liq, lrq

5.4 Output Parameters

1: **iq(liq) – int32 array**

Integer data defining the interpolant $Q(x, y)$.

2: **rq(lrq) – double array**

Real data defining the interpolant $Q(x, y)$.

3: **ifail – int32 scalar**

0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

ifail $= 1$

On entry, $\mathbf{m} < 6$,
or $0 < \mathbf{nq} < 5$,
or $\mathbf{nq} > \min(40, \mathbf{m} - 1)$,
or $\mathbf{nw} > \min(40, \mathbf{m} - 1)$,
or $\mathbf{liq} < 2 \times \mathbf{m} + 1$,
or $\mathbf{lrq} < 6 \times \mathbf{m} + 5$.

ifail $= 2$

On entry, $(\mathbf{x}(i), \mathbf{y}(i)) = (\mathbf{x}(j), \mathbf{y}(j))$ for some $i \neq j$.

ifail $= 3$

On entry, all the data points are collinear. No unique solution exists.

7 Accuracy

On successful exit, the function generated interpolates the input data exactly and has quadratic accuracy.

8 Further Comments

8.1 Timing

The time taken for a call to e01sg will depend in general on the distribution of the data points. If **x** and **y** are uniformly randomly distributed, then the time taken should be O(**m**). At worst $O(\mathbf{m}^2)$ time will be required.

8.2 Choice of N_w and N_q

Default values of the parameters N_w and N_q may be selected by calling e01sg with $\mathbf{nw} \leq 0$ and $\mathbf{nq} \leq 0$. These default values may well be satisfactory for many applications.

If nondefault values are required they must be supplied to e01sg through positive values of **nw** and **nq**. Increasing these parameters makes the method less local. This may increase the accuracy of the resulting interpolant at the expense of increased computational cost. The default values $\mathbf{nw} = \min(19, \mathbf{m} - 1)$ and $\mathbf{nq} = \min(13, \mathbf{m} - 1)$ have been chosen on the basis of experimental results reported in Renka 1988c. In these experiments the error norm was found to vary smoothly with N_w and N_q, generally increasing monotonically and slowly with distance from the optimal pair. The method is not therefore thought to be particularly sensitive to the parameter values. For further advice on the choice of these parameters see Renka 1988c.

9 Example

```
x = [11.16;
     12.85;
     19.85;
     19.72;
     15.91;
     0;
     20.87;
     3.45;
     14.26;
     17.43;
     22.8;
```

```
      7.58;
      25;
      0;
      9.66;
      5.22;
      17.25;
      25;
      12.13;
      22.23;
      11.52;
      15.2;
      7.54;
      17.32;
      2.14;
      0.51;
      22.69;
      5.47;
      21.67;
      3.31];
y = [1.24;
      3.06;
      10.72;
      1.39;
      7.74;
      20;
      20;
      12.78;
      17.87;
      3.46;
      12.39;
      1.98;
      11.87;
      0;
      20;
      14.66;
      19.57;
      3.87;
      10.79;
      6.21;
      8.529999999999999;
      0;
      10.69;
      13.78;
      15.03;
      8.369999999999999;
      19.63;
      17.13;
      14.36;
      0.33];
f = [22.15;
      22.11;
      7.97;
      16.83;
      15.3;
      34.6;
      5.74;
      41.24;
      10.74;
      18.6;
      5.47;
      29.87;
      4.4;
      58.2;
      4.73;
      40.36;
      6.43;
      8.74;
      13.71;
      10.25;
      15.74;
```

```
     21.6;
     19.31;
     12.11;
     53.1;
     49.43;
     3.25;
     28.63;
     5.52;
     44.08];
nw = int32(0);
nq = int32(0);
[iq, rq, ifail] = e01sg(x, y, f, nw, nq)
```

```
iq =
     array elided
rq =
     array elided
ifail =
           0
```